Credit Value Adjustment (Payoff-at-Maturity contracts, Equity Swaps, and Interest Rate Swaps)

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Abstract

Credit Value Adjustment estimators for several financial derivatives were developed, and typical features of CVAs are numerically investigated.

Keywords:
Credit Value Adjustment, Equity Swap, Interest Rate Swap

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1 Credit Value Adjustment (CVA) definition

The Credit Value Adjustment is based on the assumption that the value of a financial contract with a defaultable counterparty is reduced by the value of the expected default losses. Simple CVA estimators were developed for Payoff-at-Maturity contracts, for equity swaps (CONSULTING, 2013), and for interest rate swaps. General formula for CVA (Gregory, 2010) is as follows:

$$CVA = (1-\rho)\sum_{i=1}^{n} EE_i \cdot PD_i \tag{1}$$

where ρ is the recovery rate, EE_i is the Expected Exposure (EE) at time $t=0$ due to a cash flow at t_i, and PD_i is the risk-neutral Probability of Default (PD) between t_{i-1} and t_i. Time points are chosen at cash flow dates. The values of EE are calculated based on specifics of given contract type in following sections. Default probabilities for a counterparty with the known credit rating $\mathbf{R}^*$ are calculated using credit rating migration matrix $\mathbb{T}$ and credit spread curves $\mathbb{S}(R,t)$.

2 Transition matrix

Calculation of a transition matrix $\mathbb{T}$ for a given time period ΔT is necessary if available transition matrix $\mathbb{M}$ corresponds to a different time period ΔT_0. If $\Delta T = K \cdot \Delta T_0$ with $K = 1, 2, ...$ then

$$\mathbb{T} = \mathbb{M} \times \mathbb{M} \times \cdots \qquad \text{(K times)} \equiv \mathbb{M}^K \tag{2}$$

If K is not an integer (typically, it can be $\frac{1}{12}, \frac{1}{2}, 1\frac{1}{2}$, etc.) then the matrix $\mathbb{T} = \mathbb{M}^K$ according to (Israel et al., 2001) is calculated as follows:

- Using an obvious formula

  $$\mathbb{M} = e^{\log \mathbb{M}} \tag{3}$$

  we calculate the matrix-generator $\mathbb{Q} \equiv \log \mathbb{M}$:

  $$\mathbb{Q} = \sum_{k=1}^{n} (-1)^{k+1} \frac{\mathbb{D}^k}{k} \tag{4}$$

  where $\mathbb{D} = \mathbb{M} - \mathbb{I}$ (here $\mathbb{I}$ is the identity matrix). The upper limit n of the expansion is defined by the criterion

  $$\|e^{\mathbb{Q}} - \mathbb{M}\| \leq \epsilon \tag{5}$$

  where ϵ is the required accuracy.

- The required power of the matrix $\mathbb{M}$ is then calculated as $\mathbb{T} = e^{K\mathbb{Q}}$ using the following series

  $$\mathbb{T} = \mathbb{I} + \sum_{k=1}^{n} \frac{(K \cdot \mathbb{Q})^k}{k!} \tag{6}$$

3 Risk-neutral Probabilities of Default

We start from calculation of the transition matrices $\mathbb{T}(i, i+1)$ for time periods (t_i, t_{i+1}). Assuming that the historical transition matrix $\mathbb{T}$ corresponds to a time period $\Delta = t_{i+1} - t_i$ we calculate the following two matrices:

$$\begin{cases} \mathbb{T}(0,i) = \mathbb{T}^i & \text{for time period: } (0, t_i) \\ \mathbb{T}(0,i+1) = \mathbb{T}^{i+1} & \text{for time period: } (0, t_{i+1}) \end{cases} \tag{7}$$

From the credit spread curve data $\mathbb{S}(R,t)$ we obtain (by proper interpolation, if necessary) credit spreads $s_{k,i}$ corresponding to credit ratings $k \subset [1:k_m] \equiv [AAA, AA, A, BBB, BB, B, C]$ for the time period of $(0, t_i)$. Implied default probabilities are then calculated as

$$\delta_{k,i} = \frac{1 - e^{-s_{k,i} t_i}}{1 - \rho} \tag{8}$$

Next step is to modify transition matrices (7) as follows:

- Replace default probabilities (the last column) with implied default probabilities $\delta_{k,i}$
- Rescale matrix elements (except $\delta_{k,i}$) to make each row sum to be a unit

Finally, marginal transition matrices for (t_i, t_{i+1}) time periods are calculated as:

$$\mathbb{T}(i) = \mathbb{T}(0, i+1) \times \mathbb{T}(0, i)^{-1} \tag{9}$$

Using the set of transition matrices (9) we can build the probability map for rating migration of the counterparty:

$$\begin{cases} p_{k,0} = 1 & \text{for } k \text{ corresponding to the initial rating } \mathbf{R}^*, \text{ zero otherwise} \\ p_{k,i} = p_{k,i-1} \times \mathbb{T}(i) & i = 1:n \end{cases} \tag{10}$$

The probability map element $p_{k,i}$ is the probability of the counterparty to have a rating k at time t_i. Therefore, probabilities of default are:

$$PD_i = p_{k_m,i} \tag{11}$$

4 CVA for Payoff-at-Maturity contracts

4.1 CVA calculation

The Credit value adjustment calculation is straightforward in case of derivatives with payoff at maturity (vanilla options, forward rate agreements, etc.). The present value (PV) of the contract is calculated using appropriate pricer (the trinomial trees, Monte Carlo simulations, etc.). Given credit rating $\mathbf{R}^*$ of the counterparty we calculate the probability of default PD as follows. From the credit spread curve data $\mathbb{S}(R,t)$ we obtain (by proper interpolation, if necessary) the credit spread s^* corresponding to the credit rating $\mathbf{R}^*$ for the time period of $(0, T)$. Implied default probability is then calculated as

$$PD = \frac{1 - e^{-s^* T}}{1 - \rho} \tag{12}$$

Finally, we obtain the CVA value

$$CVA = (1 - \rho) \cdot PV \cdot PD^* \tag{13}$$

4.2 The CVA Calculation Examples

Let $PV = 100$ and the recovery rate $\rho = 0.5$. The credit spread data were taken as of December 2000 (see Appendix A). Results of the CVA calculation for different counterparty ratings are presented in Table 1 and in Figures (1) and (2).

Table 1: Basic CVA vs Rating

Rating	$T = 1m$	$T = 10y$
AAA	0.0297	8.2448
AA	0.0395	9.6729
A	0.0513	11.3548
BBB	0.0673	13.8592
BB	0.1583	25.6039
B	0.2178	34.8254
C	0.3321	39.1059

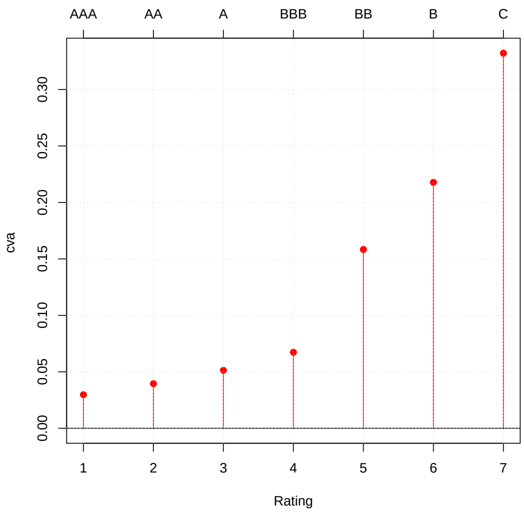

Figure 1: Maturity 1 m

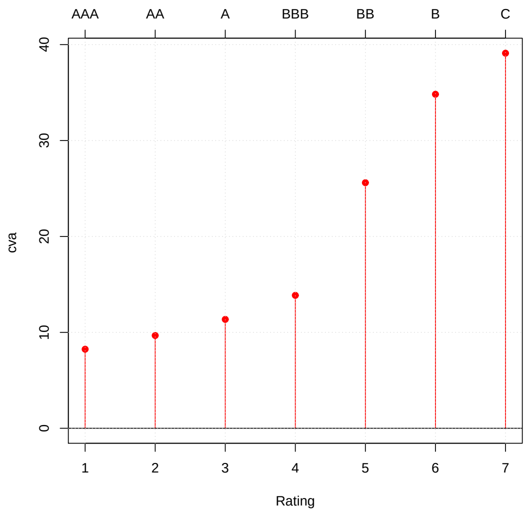

Figure 2: Maturity 10y

5 CVA for Equity Swaps

5.1 The Equity Swap Expected Exposure calculation

Terminology:

$$\begin{cases} N & \text{Notional} \\ T & \text{Maturity} \\ type & \text{Notional: fixed or float} \\ Position & \text{long or short} \\ \Delta t & \text{Swap period} \\ k_m & \text{Number of swap periods} \\ r_0 & \text{fixed payment rate} \\ dt & \text{time step} \\ r & \text{Interest rate} \\ q & \text{Dividend rate} \\ M & \text{Number of Monte Carlo scenarios} \end{cases} \tag{14}$$

5.1.1 Price tree

The price of an underlying equity can be simulated using the trinomial tree algorithm. Time axis is presented with discreet time points $t_j = (j-1)dt$, ($T = (n+1)dt$ is the option maturity, $j = 1 \cdots (n+1)$). Equity prices at time points t_j and tree nodes i are

$$\begin{aligned} S_{ji} &= S_0 \cdot u^{i-j} \\ j &= 1 \cdots (n+1) \\ i &= 1 \cdots (2j-1) \end{aligned} \tag{15}$$

The scale factors for the price moving up by u, or moving down by u^{-1} are:

$$u = e^{\sigma\sqrt{2dt}} \tag{16}$$

Probabilities for price movement up (p_u), down (p_d) or staying the same (p_m) are:

$$p_u = \frac{\sqrt{u} \cdot e^{-(r-q)dt/2} - 1}{(u-1)^2} \tag{17}$$

$$p_d = \frac{\sqrt{u} \cdot e^{-(r-q)dt/2} - u}{(u-1)^2} \tag{18}$$

$$p_m = 1 - p_u - p_d \tag{19}$$

The Equity option contract (long/short position $\pm$) cash flow (float leg) at a payment date $t_k = \Delta t \cdot k$ (the tree node (j, i)) for a given Monte Carlo scenario s is:

$$V_{ki}^{(s)} = \pm N_k^{(s)} \left(\frac{S_{ji}}{S_{j-1, i_{j-1}^{(s)}}} - 1 \right) \tag{20}$$

Where

$N_k^{(s)}$ is the notional value ($N_k^{(s)} \equiv N$ for fixed notional)

$S_{j-1, i_{j-1}^{(s)}}$ is the equity price at previous time point t_{j-1} at a previous tree node ($i_{j-1}^{(s)}$)

In case of the floating notional it is reset at time t_{j-1} to be used at t_j. Reset formula is as follows:

$$N_k = N_{k-1} \frac{S_{ji}}{S_{j-1,i^{(s)}_{j-1}}} \tag{21}$$

In the following two sections below (5.1.2, 5.1.3) the Monte Carlo process is described in details. As a result, at each payment time t_k for all tree nodes (i) the array of prices $V^{(s)}_{ki}$ is obtained (due to number of Monte Carlo paths through these nodes). Finally, averaging by s leads to cash flow values to be used in the tree pricing procedure

$$w_{ki} = \overline{V^{(s)}_{ki}} \tag{22}$$

5.1.2 Migration probability tree

For each scenario the path starts at the tree root $(1,1)$ and goes through the tree nodes switching either up $(i \to (i+2))$, down $(i \to i)$ or staying the same $(i \to (i+1))$ according to probabilities p_u, p_d, and p_m. A probability P_{ji} to reach a node (j,i) is calculated as follows:

$P_{11} = 1$, $P_{22} = p_m$

$$P_{j+1,i} = \begin{cases} P_{j,i-2}p_u, & \text{if } i = 2j+1; \quad j = 1\cdots,n \\ P_{j,i-2}p_u + P_{j,i-1}p_m, & \text{if } i = 2j; \quad j = 3\cdots,n \\ P_{ji}p_d + P_{j,i-1}p_m + P_{j,i-2}p_u, & \text{if } i = 3\cdots,2j-3; \quad j = 2\cdots,n \\ P_{j,i-1}p_m + P_{j,i}p_d, & \text{if } i = 2; \quad j = 3\cdots,n \\ P_{j,i}p_d, & \text{if } i = 1; \quad j = 1\cdots,n \end{cases} \tag{23}$$

5.1.3 Building the s^{th} Monte Carlo scenario

We start from the tree root node $(1,1)$. From each node (j,i) the path goes up or down according to the random value of ϵ

$$\begin{cases} \epsilon < p_d & \text{downward } (i \to i) \\ p_d < \epsilon < p_d + p_m & \text{same } (i \to i+1) \\ \epsilon > p_d + p_m & \text{upward } (i \to i+2) \end{cases} \tag{24}$$

where ϵ is a uniformly distributed random number $(0 < \epsilon < 1)$.
For each M.C. scenario s we add $V^{(s)}_{ki}$ to the sequence of prices for nodes (i) at payment times t_k along the Monte Carlo path. After all Monte Carlo scenarios are done we take average of s-sequences and record it as w_{ki} as in (22).

Attention! In case of not sufficient number of Monte Carlo scenarios some nodes are never reached. In these cases $w_{ki} = 0$

5.1.4 Backward Induction

Working back from $(n+1,i)$ to the tree root $(1,1)$:
At maturity we have node values $Q_{n+1,i}$:

$$Q_{n+1,i} = w_{k_m,i} \tag{25}$$

Backward induction $(j = n) \rightarrow (j = 1)$:

$$Q_{j,i} = (Q_{j+1,i} \cdot p_d + Q_{j+1,i+1} \cdot p_m + Q_{j+1,i+2} \cdot p_u)\, e^{-r \cdot dt} \quad i = 1, \cdots, 2j-1 \tag{26}$$

At each payment date t_k $(k = k_m \cdots k = 1)$ we take account of cash flows:

$$Q_{j,i} \leftarrow Q_{j,i} + w_{ki} \tag{27}$$

Finally, we obtain all node values $Q_{j,i}$ and the Equity Swap value at $t = 0$:

$$Q_{1,1} \tag{28}$$

5.1.5 Expected Exposure

The Expected Exposure is based on all non-negative values of Q_{jk} at t_j discounted to $t = 0$:

$$EE_j = \sum_{k=1}^{2j-1} \max(0, Q_{jk}) P_{jk} \cdot e^{-r \cdot t_j} \tag{29}$$

5.2 The Equity Swap Rate calculation

The Equity Swap rate r_0 can be calculated using Expected Exposure at $t = 0$ reduced by the CVA value based on the assumption that the present value $\left(EE_1^{(0)} - CVA\right)$ of the swap (receive float) is offset by future fixed rate payments. The CVA values are calculated using (1), (11), and (29). The result is:

$$r_0 = \frac{EE_1^{(0)} - CVA}{\sum\limits_{k=1}^{k_m} \Delta t \cdot e^{-rt_k}} \tag{30}$$

5.3 Test Results

For testing we use the credit spread data (Appendix A) and the $3m$ transition matrix (Appendix B).

Table 2: Parameters

Position	Long		Transition Matrix	3 m
Notional	100		Recovery rate	50 %
Maturity	6 m		Monte Carlo	50000
Swap period	3 m			
Time step	1 d			
Volatility	25 %			
Interest rate	2%			
Dividend rate	0%			

If CVA is neglected then $r_0 = 2.11\%$ and $EE_1 = 1.048$. Results of the CVA calculation are presented in Table 3. Dependencies of CVA and of the Fixed swap rates of the counterparty credit rating are presented in Figure 3 and in Figure 4.

Table 3: Equity Swap CVA and Fixed Rate

Rating	CVA	r_0, %
AAA	0.0108	2.09
AA	0.0143	2.08
A	0.0185	2.07
BBB	0.0246	2.06
BB	0.0589	1.99
B	0.0820	1.95
C	0.1228	1.86

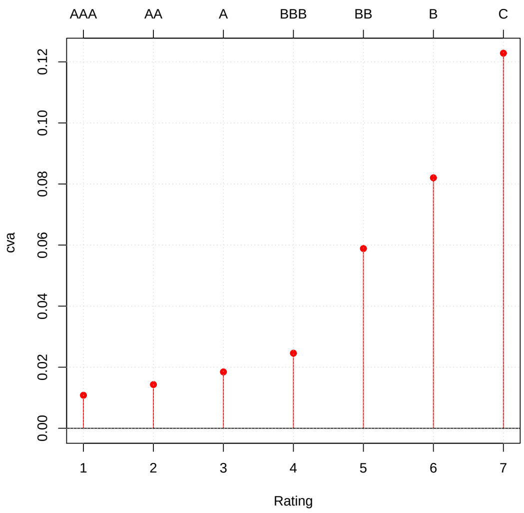

Figure 3: CVA

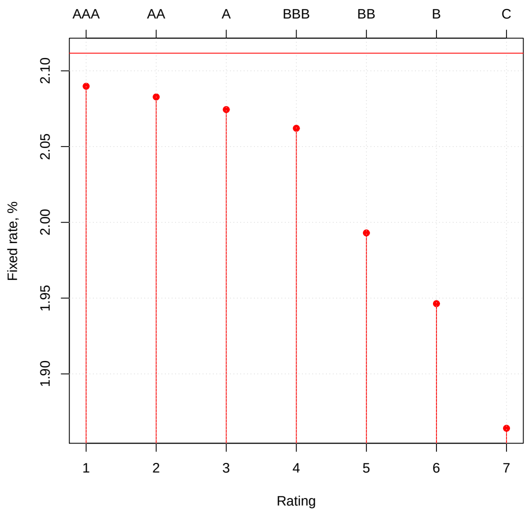

Figure 4: Swap rate

An example of the time dependence of the CVA and of the Equity swap exposure for a C-rated counterparty is presented in Figures 5 and 6.

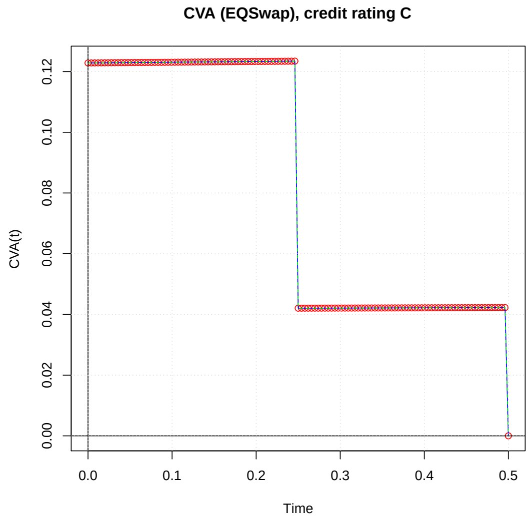

Figure 5: CVA vs time

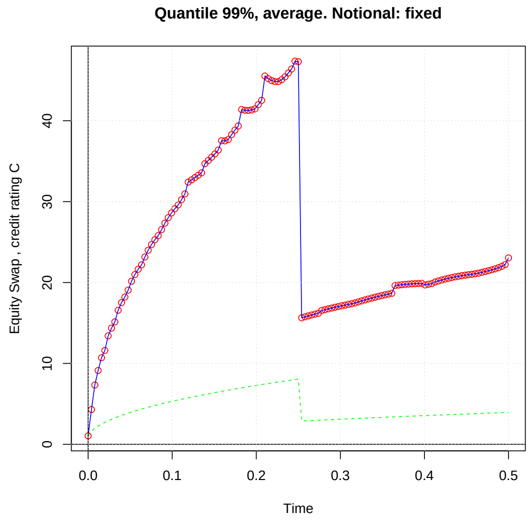

Figure 6: Equity swap exposure

6 The Credit value adjusted Swap Rate

6.1 Fair Swap rate for defaultable counterparties

Consider an Interest Rate swap contract (notional N) bought by the Bank (receive float, pay fixed) with the credit rating of R_B which is sold by a Counterparty (receive fixed, pay float) with the credit rating of R_{CP}. The present values of the float/fixed receiver is

$$\begin{cases} PV_{float} = N \cdot \sum\limits_{k=1}^{n} r_k \cdot \Delta \cdot e^{-y_k t_k} \cdot (1 - pd_i^{(CP)}) \\ PV_{fixed} = N \cdot \sum\limits_{k=1}^{n} r_f \cdot \Delta \cdot e^{-y_k t_k} \cdot (1 - pd_i^{(B)}) \end{cases} \tag{31}$$

where n is number of payments, Δ is the coverage period, r_k is the float interest rate at k^{th} payment date t_k, y_k is the discount rate at t_k, and $pd_i^{(CP)}/pd_i^{(B)}$ are default probabilities of the fixed/float receiver.
The fair fixed rate r_f can be calculated based on $PV_{float} = PV_{fixed}$ which leads to

$$r_f = \frac{\sum\limits_{k=1}^{n} r_k \cdot e^{-y_k t_k} \cdot (1 - pd_i^{(CP)})}{\sum\limits_{k=1}^{n} e^{-y_k t_k} \cdot (1 - pd_i^{(B)})} \tag{32}$$

6.2 Swap rate examples

As an example we calculate Credit value Adjusted swap rates for a typical float / fixed Interest Rate Swap with payment frequency of $3m$ using transition matrix (Appendix B), credit spread data (Appendix A), and the yield curve (Appendix C). The recovery rates for both parties are 50%. In case of a $3m$ maturity (Table 4) the no-default swap rate is 0.6250%.

Table 4: Swap Rates (%) at $T = 3m$

Counterparty		AAA	AA	A	BBB	BB	B	C
	AAA	0.6250	0.6254	0.6258	0.6264	0.6300	0.6324	0.6368
	AA	0.6246	0.6250	0.6254	0.6261	0.6296	0.6321	0.6364
	A	0.6242	0.6246	0.6250	0.6256	0.6292	0.6316	0.6360
	BBB	0.6236	0.6239	0.6244	0.6250	0.6285	0.6310	0.6353
	BB	0.6201	0.6204	0.6209	0.6215	0.6250	0.6274	0.6317
	B	0.6176	0.6180	0.6184	0.6191	0.6226	0.6250	0.6293
	C	0.6134	0.6138	0.6142	0.6149	0.6183	0.6207	0.6250

In case of a $5y$ maturity (Table 5) the no-default swap rate is 2.656%

Table 5: Swap Rates (%) at $T = 5y$

Counterparty		AAA	AA	A	BBB	BB	B	C
	AAA	2.655	2.657	2.660	2.666	2.686	2.702	2.727
	AA	2.653	2.655	2.658	2.663	2.684	2.700	2.724
	A	2.650	2.652	2.655	2.660	2.680	2.696	2.721
	BBB	2.644	2.645	2.648	2.654	2.674	2.690	2.714
	BB	2.622	2.623	2.626	2.632	2.652	2.668	2.692
	B	2.604	2.606	2.609	2.614	2.634	2.650	2.674
	C	2.579	2.580	2.583	2.589	2.608	2.624	2.648

An example of the swap rate dependence on the counterparty rating is presented in Figure 7 (horizontal line represent the no-default swap rate 4.214%).

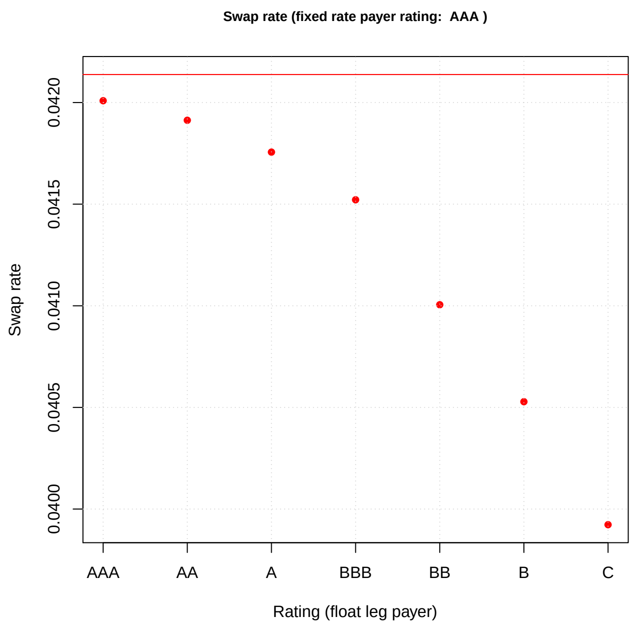

Figure 7

7 Source Codes

The source codes for CVA calculations can be ordered through Yashkir Consulting web site.

8 References

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A Credit Spread Data

Credit spread data used corresponds to December of 2000

Tenor	**AAA**	**AA**	**A**	**BBB**	**BB**	**B**	**C**
1m	0.00357	0.00474	0.00616	0.00808	0.01903	0.02619	0.03999
2m	0.00361	0.00478	0.00619	0.00815	0.01928	0.02675	0.04048
3m	0.00364	0.00482	0.00621	0.00823	0.01952	0.0273	0.04096
4m	0.00367	0.00487	0.00624	0.0083	0.01977	0.02784	0.04144
5m	0.00371	0.00491	0.00627	0.00837	0.02000	0.02837	0.04191
6m	0.00374	0.00495	0.0063	0.00844	0.02024	0.02889	0.04238
7m	0.00378	0.00500	0.00634	0.00852	0.02047	0.0294	0.04284
8m	0.00381	0.00504	0.00637	0.00859	0.0207	0.0299	0.0433
9m	0.00385	0.00509	0.0064	0.00867	0.02092	0.03039	0.04375
10m	0.00389	0.00513	0.00644	0.00874	0.02114	0.03087	0.04419
11m	0.00393	0.00518	0.00648	0.00881	0.02136	0.03135	0.04463
12m	0.00396	0.00522	0.00652	0.00889	0.02157	0.03181	0.04507
13m	0.004	0.00527	0.00656	0.00896	0.02178	0.03227	0.0455
14m	0.00404	0.00532	0.0066	0.00904	0.02198	0.03272	0.04593
15m	0.00408	0.00537	0.00665	0.00911	0.02219	0.03316	0.04635
16m	0.00412	0.00542	0.00669	0.00919	0.02238	0.03359	0.04676
17m	0.00416	0.00546	0.00674	0.00926	0.02258	0.03401	0.04717
18m	0.0042	0.00551	0.00679	0.00934	0.02277	0.03443	0.04758
19m	0.00424	0.00556	0.00684	0.00941	0.02296	0.03484	0.04798
20m	0.00429	0.00561	0.00689	0.00949	0.02315	0.03524	0.04838
21m	0.00433	0.00567	0.00694	0.00956	0.02333	0.03563	0.04877
22m	0.00437	0.00572	0.00699	0.00964	0.02351	0.03601	0.04916
23m	0.00442	0.00577	0.00704	0.00972	0.02369	0.03639	0.04954
24m	0.00446	0.00582	0.0071	0.00979	0.02386	0.03676	0.04992
25m	0.00450	0.00587	0.00715	0.00987	0.02403	0.03712	0.0503
26m	0.00455	0.00592	0.00721	0.00995	0.0242	0.03748	0.05067
27m	0.00459	0.00598	0.00727	0.01002	0.02437	0.03783	0.05104
28m	0.00464	0.00603	0.00733	0.0101	0.02453	0.03817	0.0514
29m	0.00468	0.00608	0.00738	0.01017	0.02469	0.03851	0.05176
30m	0.00473	0.00614	0.00745	0.01025	0.02485	0.03884	0.05211
31m	0.00477	0.00619	0.00751	0.01033	0.025	0.03916	0.05246
32m	0.00482	0.00625	0.00757	0.0104	0.02515	0.03948	0.05281
33m	0.00487	0.0063	0.00763	0.01048	0.0253	0.03979	0.05315
34m	0.00491	0.00636	0.00769	0.01056	0.02545	0.04010	0.05349
35m	0.00496	0.00641	0.00776	0.01063	0.0256	0.04040	0.05383
36m	0.00501	0.00647	0.00782	0.01071	0.02574	0.04070	0.05416

37m	0.00506	0.00652	0.00789	0.01079	0.02588	0.04099	0.05449
38m	0.0051	0.00658	0.00795	0.01086	0.02602	0.04127	0.05482
39m	0.00515	0.00664	0.00802	0.01094	0.02616	0.04155	0.05514
40m	0.0052	0.00669	0.00809	0.01102	0.0263	0.04182	0.05546
41m	0.00525	0.00675	0.00816	0.01109	0.02643	0.04209	0.05578
42m	0.0053	0.0068	0.00823	0.01117	0.02656	0.04236	0.05609
43m	0.00535	0.00686	0.00829	0.01124	0.02669	0.04262	0.0564
44m	0.0054	0.00692	0.00836	0.01132	0.02682	0.04287	0.0567
45m	0.00545	0.00697	0.00843	0.0114	0.02695	0.04312	0.05701
46m	0.0055	0.00703	0.0085	0.01147	0.02707	0.04337	0.05731
47m	0.00555	0.00709	0.00858	0.01155	0.02719	0.04362	0.05761
48m	0.0056	0.00715	0.00865	0.01162	0.02732	0.04385	0.0579
49m	0.00565	0.0072	0.00872	0.0117	0.02744	0.04409	0.0582
50m	0.0057	0.00726	0.00879	0.01178	0.02756	0.04432	0.05849
51m	0.00575	0.00732	0.00886	0.01185	0.02767	0.04455	0.05877
52m	0.0058	0.00738	0.00893	0.01193	0.02779	0.04477	0.05906
53m	0.00585	0.00743	0.00901	0.012	0.02791	0.045	0.05934
54m	0.0059	0.00749	0.00908	0.01208	0.02802	0.04522	0.05962
55m	0.00595	0.00755	0.00915	0.01215	0.02813	0.04543	0.0599
56m	0.00600	0.00761	0.00923	0.01223	0.02824	0.04564	0.06017
57m	0.00605	0.00766	0.00930	0.0123	0.02835	0.04585	0.06045
58m	0.00610	0.00772	0.00937	0.01237	0.02846	0.04606	0.06072
59m	0.00615	0.00778	0.00945	0.01245	0.02857	0.04627	0.06099
60m	0.00620	0.00784	0.00952	0.01252	0.02868	0.04647	0.06126
61m	0.00626	0.00789	0.00959	0.0126	0.02879	0.04667	0.06152
62m	0.00631	0.00795	0.00967	0.01267	0.02890	0.04687	0.06179
63m	0.00636	0.00801	0.00974	0.01274	0.02900	0.04707	0.06205
64m	0.00641	0.00807	0.00981	0.01281	0.02911	0.04726	0.06231
65m	0.00646	0.00812	0.00989	0.01289	0.02921	0.04746	0.06257
66m	0.00651	0.00818	0.00996	0.01296	0.02932	0.04765	0.06282
67m	0.00656	0.00824	0.01003	0.01303	0.02942	0.04784	0.06308
68m	0.00662	0.00829	0.0101	0.0131	0.02953	0.04803	0.06333
69m	0.00667	0.00835	0.01018	0.01317	0.02963	0.04822	0.06359
70m	0.00672	0.00841	0.01025	0.01325	0.02973	0.0484	0.06384
71m	0.00677	0.00846	0.01032	0.01332	0.02984	0.04859	0.06409
72m	0.00682	0.00852	0.01039	0.01339	0.02994	0.04878	0.06434
73m	0.00687	0.00857	0.01046	0.01346	0.03004	0.04896	0.06458
74m	0.00692	0.00863	0.01053	0.01353	0.03014	0.04915	0.06483
75m	0.00697	0.00868	0.0106	0.0136	0.03025	0.04933	0.06508
76m	0.00702	0.00874	0.01067	0.01367	0.03035	0.04952	0.06532
77m	0.00707	0.00879	0.01074	0.01373	0.03045	0.0497	0.06557
78m	0.00712	0.00885	0.01081	0.0138	0.03056	0.04989	0.06581
79m	0.00717	0.0089	0.01088	0.01387	0.03066	0.05007	0.06605
80m	0.00723	0.00896	0.01095	0.01394	0.03077	0.05026	0.06629
81m	0.00728	0.00901	0.01101	0.01401	0.03087	0.05045	0.06654
82m	0.00732	0.00906	0.01108	0.01407	0.03097	0.05063	0.06678
83m	0.00737	0.00912	0.01115	0.01414	0.03108	0.05082	0.06702
84m	0.00742	0.00917	0.01121	0.01421	0.03119	0.05101	0.06726
85m	0.00747	0.00922	0.01128	0.01427	0.03129	0.0512	0.0675
86m	0.00752	0.00928	0.01134	0.01434	0.0314	0.05139	0.06774
87m	0.00757	0.00933	0.0114	0.0144	0.03151	0.05159	0.06797
88m	0.00762	0.00938	0.01147	0.01447	0.03162	0.05178	0.06821
89m	0.00767	0.00943	0.01153	0.01453	0.03173	0.05198	0.06845
90m	0.00772	0.00948	0.01159	0.01459	0.03184	0.05217	0.06869
91m	0.00777	0.00953	0.01165	0.01466	0.03195	0.05237	0.06893
92m	0.00781	0.00958	0.01171	0.01472	0.03206	0.05258	0.06917
93m	0.00786	0.00963	0.01177	0.01478	0.03218	0.05278	0.06941
94m	0.00791	0.00968	0.01182	0.01484	0.03229	0.05299	0.06965
95m	0.00795	0.00973	0.01188	0.0149	0.03241	0.0532	0.06989
96m	0.008	0.00977	0.01193	0.01496	0.03253	0.05341	0.07013
97m	0.00805	0.00982	0.01199	0.01502	0.03264	0.05362	0.07037
98m	0.00809	0.00987	0.01204	0.01508	0.03276	0.05384	0.07061

99m	0.00814	0.00991	0.01209	0.01514	0.03289	0.05406	0.07085
100m	0.00819	0.00996	0.01214	0.0152	0.03301	0.05428	0.0711
101m	0.00823	0.01001	0.01219	0.01526	0.03313	0.05451	0.07134
102m	0.00827	0.01005	0.01224	0.01531	0.03326	0.05474	0.07158
103m	0.00832	0.0101	0.01229	0.01537	0.03339	0.05497	0.07183
104m	0.00836	0.01014	0.01234	0.01543	0.03352	0.05521	0.07208
105m	0.00841	0.01018	0.01238	0.01548	0.03365	0.05545	0.07232
106m	0.00845	0.01022	0.01242	0.01554	0.03378	0.0557	0.07257
107m	0.00849	0.01027	0.01247	0.01559	0.03392	0.05594	0.07282
108m	0.00854	0.01031	0.01251	0.01564	0.03406	0.0562	0.07307
109m	0.00858	0.01035	0.01255	0.0157	0.03419	0.05646	0.07332
110m	0.00862	0.01039	0.01258	0.01575	0.03434	0.05672	0.07357
111m	0.00866	0.01043	0.01262	0.0158	0.03448	0.05699	0.07383
112m	0.0087	0.01047	0.01266	0.01585	0.03463	0.05726	0.07408
113m	0.00874	0.01051	0.01269	0.0159	0.03477	0.05753	0.07434
114m	0.00878	0.01054	0.01272	0.01595	0.03492	0.05782	0.0746
115m	0.00882	0.01058	0.01275	0.016	0.03508	0.0581	0.07486
116m	0.00886	0.01062	0.01278	0.01605	0.03523	0.05839	0.07512
117m	0.0089	0.01065	0.01281	0.01609	0.03539	0.05869	0.07539
118m	0.00894	0.01069	0.01283	0.01614	0.03555	0.059	0.07565
119m	0.00897	0.01072	0.01286	0.01619	0.03571	0.05931	0.07592
120m	0.00901	0.01075	0.01288	0.01623	0.03588	0.05962	0.07619

B Transition Matrix

Table 7: 3 m Transition matrix (December 2000)

	AAA	AA	A	BBB	BB	B	C	D
AAA	0.8892	0.0926	0.0093	0.0059	0.0009	0.0019	0.0001	0.0000
AA	0.0931	0.7663	0.1331	0.0027	0.0035	0.0008	0.0005	0.0000
A	0.0000	0.1097	0.7588	0.1190	0.0105	0.0029	0.0001	0.0002
BBB	0.0026	0.0000	0.1243	0.7453	0.1202	0.0076	0.0020	0.0004
BB	0.0000	0.0033	0.0000	0.1233	0.7604	0.1136	0.0000	0.0017
B	0.0001	0.0000	0.0026	0.0000	0.0974	0.8205	0.0777	0.0026
C	0.0000	0.0001	0.0000	0.0015	0.0000	0.0673	0.9028	0.0287
D	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000

C Yield Curve

Table 8: Example of a yield curve

tenor, y	yield rate, %
0	0.5
1	1
2	1.5
3	2
5	2.7
7	3.2
10	4.5
30	6