FLEX Option Price Calculator (modified Monte Carlo trinomial tree)

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Contents

1 Methodology

Terminology:

$$\begin{cases} C_D & \text{Domestic currency} \\ C_f & \text{Foreign currency} \\ FX[C_D/C_f] & \text{Exchange rate (Foreign-to-Domestic currency conversion)} \end{cases} \tag{1}$$

Throughout this paper the value F refers to $FX[C_D/C_f]$, and F^{-1} refers to $FX[C_f/C_D]$. The FLEX contract payoff at maturity T is one of:

$$V_T = \begin{cases} N\left[1-(F^{-1})^{(\tau_1)}F_T\right]^+ & \text{Pay } C_f \text{ Receive } C_D \\ N\left[-1+(F^{-1})^{(\tau_2)}F_T\right]^+ & \text{Pay } C_D \text{ Receive } C_f \\ N\left[(F^{-1})^{(\tau_1)}-(F^{-1})^{(\tau_2)}\right]^+ F_T & \text{Pay } C_f \text{ Receive } C_f \end{cases} \tag{2}$$

Where

$[\cdot]^+ \equiv \max[0,\cdot]$

$(\cdot)^{\tau_k}$ means averaging during the time period of $[t_{k1}\cdots t_{k2}]$

N is the notional amount

1.1 Price tree

The price of an underlying FX rate can be simulated using the trinomial tree algorithm. Time axis is presented with discreet time points $t_j = (j-1)\cdot\Delta t$ (Δt is the time step and $T = (n+1)\cdot\Delta t$ is the option maturity, $j = 1\cdots(n+1)$). FX rates at time points t_j and tree nodes i are

$$F_{ji} = F_0 \cdot u^{i-j} \quad (3)$$
$$j = 1\cdots(n+1)$$
$$i = 1\cdots(2j-1)$$

The scale factors for FX moving up by u, or moving down by u^{-1} are:

$$u = e^{\sigma\sqrt{2\Delta t}} \quad (4)$$

Probabilities for price movement up (p_u), down (p_d) or staying the same (p_m) are:

$$p_u = \frac{\sqrt{u}\cdot e^{-(r_D-r_f)\Delta t/2} - 1}{(u-1)^2} \quad (5)$$

$$p_d = \frac{\sqrt{u}\cdot e^{-(r_D-r_f)\Delta t/2} - u}{(u-1)^2} \quad (6)$$

$$p_m = 1 - p_u - p_d \quad (7)$$

where r_D and r_f are interest rates for domestic and foreign currencies respectively.

1.2 Migration probability tree

For each scenario the path starts at the tree root $(1,1)$ and goes through the tree nodes switching either up ($i \to (i+2)$), down ($i \to i$) or staying the same ($i \to (i+1)$) according to probabilities p_u, p_d, and p_m. A probability P_{ji} to reach a node (j,i) is calculated as follows:

$P_{11} = 1$, $P_{22} = p_m$

$$P_{j+1,i} = \begin{cases} P_{j,i-2}p_u, & \text{if } i = 2j+1;\quad j = 1\cdots,n \\ P_{j,i-2}p_u + P_{j,i-1}p_m, & \text{if } i = 2j;\quad j = 3\cdots,n \\ P_{ji}p_d + P_{j,i-1}p_m + P_{j,i-2}p_u, & \text{if } i = 3\cdots,2j-3;\quad j = 2\cdots,n \\ P_{j,i-1}p_m + P_{j,i}p_d, & \text{if } i = 2;\quad j = 3\cdots,n \\ P_{j,i}p_d, & \text{if } i = 1;\quad j = 1\cdots,n \end{cases} \quad (8)$$

1.3 Building the s^{th} Monte Carlo scenario

We start from the tree root node $(1,1)$. From each node (j,i) the path goes up or down according to the random value of ϵ

$$\begin{cases} \epsilon < p_d & \text{downward } (i \to i) \\ p_d < \epsilon < p_d + p_m & \text{same } (i \to i+1) \\ \epsilon > p_d + p_m & \text{upward } (i \to i+2) \end{cases} \quad (9)$$

where ϵ is a uniformly distributed random number ($0 < \epsilon < 1$).
For each M.C. scenario s we fill sequence of FX rates F^{-1}_{j,i_s} along the path on the time range τ_1 and its average is recorded as w_{si} (here i correspond to the end node $(n+1,i)$ of the s^{th} path). As a result (after running all M Monte Carlo scenarios), we obtain average F^{-1} for each end node $(n+1,i)$ as average by s

$$A_i^{(\tau)} = \overline{w_{s,i}} \tag{10}$$

Attention! In case of not sufficient number of Monte Carlo scenarios some end nodes $(n+1,i)$ are never reached. Values of A_i for such nodes can be obtained by interpolation. It helps that values of $A_{n+1,2n-1}$ (the upper limit tree branch) and $A_{n+1,1}$ (the lower limit tree branch) can be calculated directly because in both cases the path is unique.

1.4 Payoff functions

- Receive notional N in domestic currency, pay notional in foreign currency $N \cdot F^{-1}$ applying corresponding average FX (10).The payoff value (node i) is

$$V_{n+1,i} = \mathbf{max}\left(0, \quad N\left[1 - A_i^{(\tau_1)} \cdot F_{n+1,i}\right]\right) \tag{11}$$

- Pay notional N in domestic currency, receive notional in foreign currency $N \cdot F^{-1}$ applying corresponding average FX (10).The payoff value (node i) is

$$V_{n+1,i} = \mathbf{max}\left(0, \quad N\left[-1 + A_i^{(\tau_2)} \cdot F_{n+1,i}\right]\right) \tag{12}$$

- Receive difference between notional value N in foreign currency with FX obtained by averaging in the τ_1 range and the notional value N in foreign currency with FX obtained by averaging in the τ_2 range.The payoff value (node i) is

$$V_{n+1,i} = \mathbf{max}\left(0, \quad N\left[A_i^{(\tau_1)} - A_i^{(\tau_2)}\right] F_{n+1,i}\right) \tag{13}$$

1.5 Backward Induction

Working back from $(n+1,i)$ to the tree root $(1,1)$

$$\begin{aligned} V_{j,i} &= \left(V_{j+1,i} \cdot p_d + V_{j+1,i+1} \cdot p_m + V_{j+1,i+2} \cdot p_u\right) e^{-r_D \Delta t} \\ i &= 1, \cdots, 2j-1 \end{aligned} \tag{14}$$

we arrive obtain all node values and the option value at $t = 0$:

$$V_{1,1} \tag{15}$$

1.6 Potential Future Exposure

The node values V_{ji} obtained in the previous section 1.5 and the tree probability map (section 1.2) can be used for calculation of the Potential Future Exposure (PFE). The node values V_{ji} at time t_j are associated with probabilities P_{ji} of reaching nodes (j,i). An array of pairs $[V_{ji}, P_{ji}]$ is sorted by V_{ji} (in ascending order)

$$[V_{ji}, P_{ji}] \quad \xrightarrow[sorting]{} \quad [\hat{V}_{ji}, \hat{P}_{ji}] \tag{16}$$

Cumulative probability can be calculated now as

$$C_i = \sum_{k=1}^{i} P_{jk} \tag{17}$$

Given percentile η the corresponding PFE is calculated as follows

$$\begin{aligned} \hat{i} : C_{\hat{i}} \leq \eta < C_{\hat{i}+1} \\ V_j^{\eta} = V_{j,\hat{i}} + \frac{\eta - P_{j,\hat{i}}}{P_{j,\hat{i}+1} - P_{j,\hat{i}}} (V_{j,\hat{i}+1} - V_{j,\hat{i}}) \\ PFE_j = V_j^{\eta} e^{-r \cdot t_j} \end{aligned} \tag{18}$$

1.7 Expected Exposure

The Expected Exposure is simply expected value of V_{ji}:

$$EE_j = \sum_{k=1}^{2j-1} V_{jk} P_{jk} \cdot e^{-r \cdot t_j} \tag{19}$$

2 Examples

Table 1: Parameters

Domestic currency	USD
Foreign currency	CAD
Spot FX	0.9943 (USD/CAD)
Notional, USD	100
Maturity	3 m
Time step	1 d
FX volatility	0.5 %
Interest rate (USD)	0.2731 %
Interest rate (CAD)	1.174 %
Monte Carlo scenarios	10000
Confidence level	95 %

Table 2: Results

Averaging period	Pay CAD \ Receive USD	Pay USD \ Receive CAD
2.5m to 3m	0.0248	0.00250
2m to 3m	0.0445	0.00458
1.5m to 3m	0.0652	0.00686
1m to 3m	0.0878	0.00895
0.5m to 3m	0.1089	0.01056
0 to 3m	0.1269	0.01281

Table 3: Pay CAD \ Receive CAD

Averaging period (receive)	Averaging period (pay)	Option value
0 m	3m	0.02454
0 m to 0.5 m	2.5 m to 3m	0.02051
0 m to 1 m	2 m to 3m	0.01656
0 m to 1.5 m	1.5 m to 3 m	0.01205
1 m to 2 m	2 m to 3 m	0.00836
2 m to 2.5 m	2.5 m to 3 m	0.00412

The PFE profile for $T = 3m$, Pay USD \ Receive CAD, averaging: $2.5m \rightarrow 3m$ is presented in Figure 1.

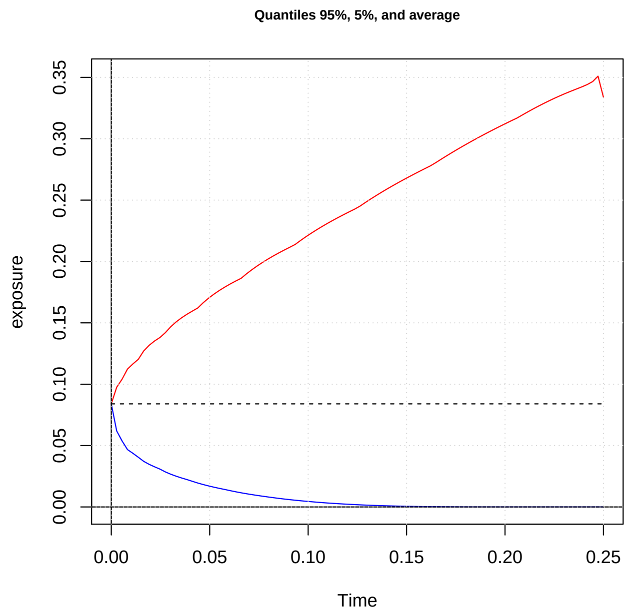

Figure 1: FLEX Option PFE

Dependence of the average FX on the FX at maturity is presented in figure 2 (red). In the same graph the probability distribution at maturity is also presented (blue).

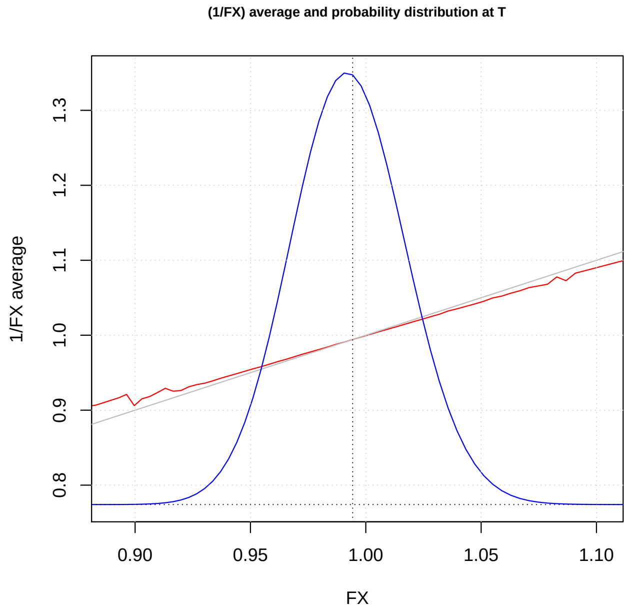

Figure 2: FX average vs FX_T