

Risk-neutral Transition Matrix Calculator

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1 The Method

We start from the transition matrix \mathbf{T}_{kk} obtained from historical data for a given time period τ_h . Here index k corresponds to the credit rating R_k ($k = 1 \dots k_m$). For simplicity we consider the following set of credit ratings

$$\vec{R} = \{AAA, AA, A, BBB, BB, B, C, D\} \quad (1)$$

This matrix reflects averaged historical probabilities or rating migration including default probabilities. The task is to estimate a (risk-neutral) transition matrix for some future time period $(t, t + \tau)$ assuming that default probabilities for this time period are linked to the market-dependent credit spreads $S_{k,t}$ (corresponding to the period of $(0, t)$). We start from transition matrices $\mathbf{M}^{(t)}$ and $\mathbf{M}^{(t+\tau)}$ corresponding to time periods of $(0, t)$ and $(0, t + \tau)$. They are calculated as powers of the historical transition matrix using the following ratios

$$\begin{aligned} \alpha &= \frac{t}{\tau_h} \\ \beta &= \frac{t + \tau}{\tau_h} \end{aligned} \quad (2)$$

$$\begin{aligned} \mathbf{M}^{(t)} &= \mathbf{T}^\alpha \\ \mathbf{M}^{(t+\tau)} &= \mathbf{T}^\beta \end{aligned} \quad (3)$$

The method of calculating matrix power (which can be any non-negative rational number) is presented in (CONSULTING).

Given credit spreads $S_{k,t}$ we calculate default probabilities at t and $t + \tau$ as

$$\begin{aligned} \delta_{k,t} &= \frac{1 - e^{-S_{k,t} \cdot t}}{1 - \rho} \\ \delta_{k,t+\tau} &= \frac{1 - e^{-S_{k,t+\tau} \cdot (t+\tau)}}{1 - \rho} \end{aligned} \quad (4)$$

Here ρ is the recovery rate parameter.

The risk neutral transition matrices $\mathbf{R}^{(t'')}$ for time periods of $(0, t'')$ ($t'' = t$ and $t'' = t + \tau$) is then calculated as:

$$R_{k, k_m}^{(t'')} = \delta_{k, t''}$$

$$R_{k, k'}^{(t'')} = M_{k, k'}^{(t'')} \frac{1 - \delta_{k, t''}}{\sum_{p=1}^{k_m-1} M_{k, p}^{(t'')}} \quad (5)$$

$$k = 1 \dots (k_m - 1); \quad k' = 1 \dots (k_m - 1)$$

Finally, we calculate marginal risk neutral transition matrix $\hat{M}^{(t)}$ which correspond to credit rating migration in the time interval $(t, t + \tau)$:

$$\hat{\mathbf{M}}^{(t)} = \mathbf{R}^{(t+\tau)} \times \left(\mathbf{R}^{(t)} \right)^{-1} \quad (6)$$

Inverse matrix calculation results sometimes in appearing of small negative elements. We replace those with zeroes. Default probabilities (\hat{M}_{k, k_m}) may also require adjustment: if $\hat{M}_{k, k_m} < \hat{M}_{k-1, k_m}$ then

$$\hat{M}_{k, k_m} = \frac{1}{2} \left(\hat{M}_{k-1, k_m} + \hat{M}_{k+1, k_m} \right) \quad (7)$$

Such a correction requires renormalization of the matrix:

$$\hat{M}_{k, k'}^{(i)} \leftarrow \hat{M}_{k, k'}^{(i)} \frac{1 - \hat{M}_{k, k_m}^{(i)}}{\sum_{k'=1}^{k_m-1} \hat{M}_{k, k'}^{(i)}} \quad (8)$$

2 The Calculator

The algorithm presented in the previous section has been implemented in the application `rn_matrix.R` using statistical package `R`. This application requires the `R` package and the libraries `tcltk`, `base`, and `Hmisc` to be installed. Usage of the calculator is quite simple. The calculator interface is presented in Figure 1.

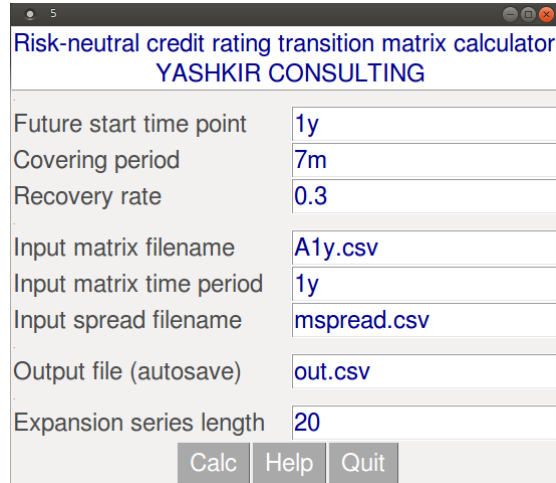


Figure 1: Risk-neutral Transition Matrix Calculator interface

User can modify input parameters and run the calculator. The calculation result (risk-neutral matrix) is then saved in the output file (see Table 1) and is displayed as in Figure 2.

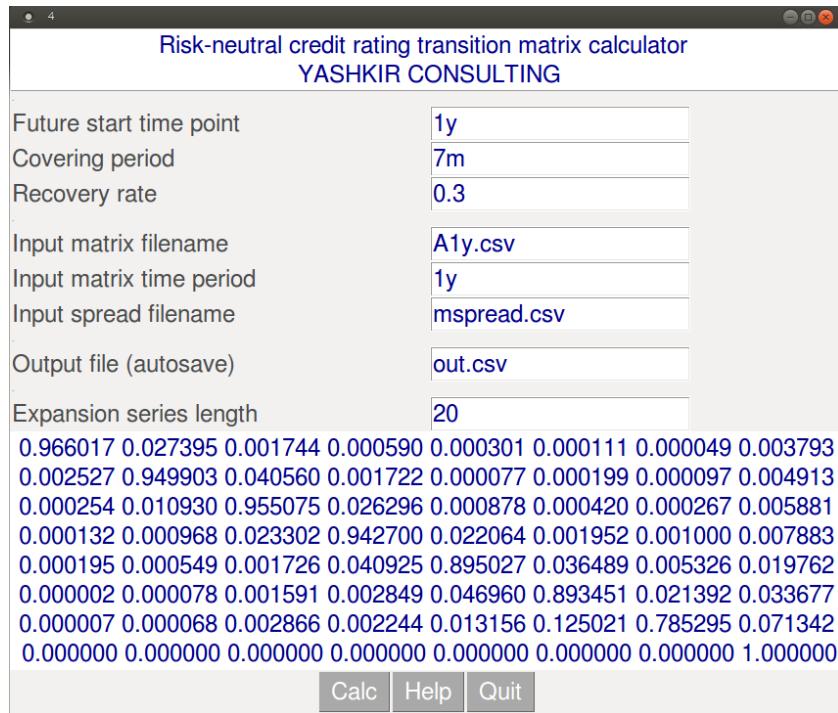


Figure 2: Calculation result

Table 1: The output file content

	,AAA	,AA	,A	,BBB	,BB	,B	,C	,D
AAA	,0.967178	,0.02371	,0.001445	,0.000512	,0.000274	,9.8E-05	,4E-05	,0.006743
AA	,0.002154	,0.952798	,0.035057	,0.001433	,6.5E-05	,0.000176	,7.7E-05	,0.00824
A	,0.000213	,0.009333	,0.956624	,0.022798	,0.000753	,0.000366	,0.000212	,0.009701
BBB	,0.00011	,0.000803	,0.019848	,0.946129	,0.019822	,0.001656	,0.000789	,0.010843
BB	,0.000158	,0.000445	,0.001353	,0.033962	,0.905165	,0.030989	,0.004056	,0.023872
B	,1E-06	,6.1E-05	,0.001306	,0.002251	,0.041013	,0.893078	,0.016619	,0.04567
C	,6E-06	,5.5E-05	,0.002459	,0.001896	,0.011544	,0.111844	,0.744185	,0.128011
D	,0	,0	,0	,0	,0	,0	,0	,1

Test results were obtained with the input data files presented in Appendix A.

A Appendix

The historical transition matrix **A1y.csv** is presented in the Table 2 and the credit spread data **mspread.csv** is presented in the Table 3.

Table 2: The 1-year historical transition matrix

AAA	AA	A	BBB	BB	B	C	D
0.948692	0.045796	0.003618	0.001044	0.000500	0.000200	0.000100	0.000050
0.004249	0.923669	0.067663	0.003524	0.000176	0.000353	0.000198	0.000167
0.000447	0.018313	0.933776	0.043805	0.001798	0.000774	0.000543	0.000544
0.000230	0.001789	0.039034	0.915315	0.035780	0.003859	0.002043	0.001950
0.000333	0.000960	0.003561	0.068128	0.846176	0.061182	0.010828	0.008832
0.000010	0.000152	0.002643	0.005722	0.072033	0.823130	0.038742	0.057568
0.000010	0.000100	0.003333	0.002938	0.016956	0.138900	0.579481	0.258283
0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000

	AAA	AA	A	BBB	BB	B	C
1m	0.00357	0.00474	0.00616	0.00808	0.01903	0.02619	0.03999
2m	0.00361	0.00478	0.00619	0.00815	0.01928	0.02675	0.04048
3m	0.00364	0.00482	0.00621	0.00823	0.01952	0.0273	0.04096
4m	0.00367	0.00487	0.00624	0.0083	0.01977	0.02784	0.04144
5m	0.00371	0.00491	0.00627	0.00837	0.02	0.02837	0.04191
6m	0.00374	0.00495	0.0063	0.00844	0.02024	0.02889	0.04238
7m	0.00378	0.005	0.00634	0.00852	0.02047	0.0294	0.04284
8m	0.00381	0.00504	0.00637	0.00859	0.0207	0.0299	0.0433
9m	0.00385	0.00509	0.0064	0.00867	0.02092	0.03039	0.04375
10m	0.00389	0.00513	0.00644	0.00874	0.02114	0.03087	0.04419
11m	0.00393	0.00518	0.00648	0.00881	0.02136	0.03135	0.04463
12m	0.00396	0.00522	0.00652	0.00889	0.02157	0.03181	0.04507
13m	0.004	0.00527	0.00656	0.00896	0.02178	0.03227	0.0455
14m	0.00404	0.00532	0.0066	0.00904	0.02198	0.03272	0.04593
15m	0.00408	0.00537	0.00665	0.00911	0.02219	0.03316	0.04635
16m	0.00412	0.00542	0.00669	0.00919	0.02238	0.03359	0.04676
17m	0.00416	0.00546	0.00674	0.00926	0.02258	0.03401	0.04717
18m	0.0042	0.00551	0.00679	0.00934	0.02277	0.03443	0.04758
19m	0.00424	0.00556	0.00684	0.00941	0.02296	0.03484	0.04798
20m	0.00429	0.00561	0.00689	0.00949	0.02315	0.03524	0.04838
21m	0.00433	0.00567	0.00694	0.00956	0.02333	0.03563	0.04877
22m	0.00437	0.00572	0.00699	0.00964	0.02351	0.03601	0.04916
23m	0.00442	0.00577	0.00704	0.00972	0.02369	0.03639	0.04954
24m	0.00446	0.00582	0.0071	0.00979	0.02386	0.03676	0.04992
25m	0.0045	0.00587	0.00715	0.00987	0.02403	0.03712	0.0503
26m	0.00455	0.00592	0.00721	0.00995	0.0242	0.03748	0.05067
27m	0.00459	0.00598	0.00727	0.01002	0.02437	0.03783	0.05104
28m	0.00464	0.00603	0.00733	0.0101	0.02453	0.03817	0.0514
29m	0.00468	0.00608	0.00738	0.01017	0.02469	0.03851	0.05176
30m	0.00473	0.00614	0.00745	0.01025	0.02485	0.03884	0.05211
31m	0.00477	0.00619	0.00751	0.01033	0.025	0.03916	0.05246
32m	0.00482	0.00625	0.00757	0.0104	0.02515	0.03948	0.05281
33m	0.00487	0.0063	0.00763	0.01048	0.0253	0.03979	0.05315
34m	0.00491	0.00636	0.00769	0.01056	0.02545	0.0401	0.05349
35m	0.00496	0.00641	0.00776	0.01063	0.0256	0.0404	0.05383
36m	0.00501	0.00647	0.00782	0.01071	0.02574	0.0407	0.05416

37m	0.00506	0.00652	0.00789	0.01079	0.02588	0.04099	0.05449
38m	0.0051	0.00658	0.00795	0.01086	0.02602	0.04127	0.05482
39m	0.00515	0.00664	0.00802	0.01094	0.02616	0.04155	0.05514
40m	0.0052	0.00669	0.00809	0.01102	0.0263	0.04182	0.05546
41m	0.00525	0.00675	0.00816	0.01109	0.02643	0.04209	0.05578
42m	0.0053	0.0068	0.00823	0.01117	0.02656	0.04236	0.05609
43m	0.00535	0.00686	0.00829	0.01124	0.02669	0.04262	0.0564
44m	0.0054	0.00692	0.00836	0.01132	0.02682	0.04287	0.0567
45m	0.00545	0.00697	0.00843	0.0114	0.02695	0.04312	0.05701
46m	0.0055	0.00703	0.0085	0.01147	0.02707	0.04337	0.05731
47m	0.00555	0.00709	0.00858	0.01155	0.02719	0.04362	0.05761
48m	0.0056	0.00715	0.00865	0.01162	0.02732	0.04385	0.0579
49m	0.00565	0.0072	0.00872	0.0117	0.02744	0.04409	0.0582
50m	0.0057	0.00726	0.00879	0.01178	0.02756	0.04432	0.05849
51m	0.00575	0.00732	0.00886	0.01185	0.02767	0.04455	0.05877
52m	0.0058	0.00738	0.00893	0.01193	0.02779	0.04477	0.05906
53m	0.00585	0.00743	0.00901	0.012	0.02791	0.045	0.05934
54m	0.0059	0.00749	0.00908	0.01208	0.02802	0.04522	0.05962
55m	0.00595	0.00755	0.00915	0.01215	0.02813	0.04543	0.0599
56m	0.006	0.00761	0.00923	0.01223	0.02824	0.04564	0.06017
57m	0.00605	0.00766	0.0093	0.0123	0.02835	0.04585	0.06045
58m	0.0061	0.00772	0.00937	0.01237	0.02846	0.04606	0.06072
59m	0.00615	0.00778	0.00945	0.01245	0.02857	0.04627	0.06099
60m	0.0062	0.00784	0.00952	0.01252	0.02868	0.04647	0.06126
61m	0.00626	0.00789	0.00959	0.0126	0.02879	0.04667	0.06152
62m	0.00631	0.00795	0.00967	0.01267	0.0289	0.04687	0.06179
63m	0.00636	0.00801	0.00974	0.01274	0.029	0.04707	0.06205
64m	0.00641	0.00807	0.00981	0.01281	0.02911	0.04726	0.06231
65m	0.00646	0.00812	0.00989	0.01289	0.02921	0.04746	0.06257
66m	0.00651	0.00818	0.00996	0.01296	0.02932	0.04765	0.06282
67m	0.00656	0.00824	0.01003	0.01303	0.02942	0.04784	0.06308
68m	0.00662	0.00829	0.0101	0.0131	0.02953	0.04803	0.06333
69m	0.00667	0.00835	0.01018	0.01317	0.02963	0.04822	0.06359
70m	0.00672	0.00841	0.01025	0.01325	0.02973	0.0484	0.06384
71m	0.00677	0.00846	0.01032	0.01332	0.02984	0.04859	0.06409
72m	0.00682	0.00852	0.01039	0.01339	0.02994	0.04878	0.06434
73m	0.00687	0.00857	0.01046	0.01346	0.03004	0.04896	0.06458
74m	0.00692	0.00863	0.01053	0.01353	0.03014	0.04915	0.06483
75m	0.00697	0.00868	0.0106	0.0136	0.03025	0.04933	0.06508
76m	0.00702	0.00874	0.01067	0.01367	0.03035	0.04952	0.06532
77m	0.00707	0.00879	0.01074	0.01373	0.03045	0.0497	0.06557
78m	0.00712	0.00885	0.01081	0.0138	0.03056	0.04989	0.06581
79m	0.00717	0.0089	0.01088	0.01387	0.03066	0.05007	0.06605
80m	0.00723	0.00896	0.01095	0.01394	0.03077	0.05026	0.06629
81m	0.00728	0.00901	0.01101	0.01401	0.03087	0.05045	0.06654
82m	0.00732	0.00906	0.01108	0.01407	0.03097	0.05063	0.06678
83m	0.00737	0.00912	0.01115	0.01414	0.03108	0.05082	0.06702
84m	0.00742	0.00917	0.01121	0.01421	0.03119	0.05101	0.06726
85m	0.00747	0.00922	0.01128	0.01427	0.03129	0.0512	0.0675

86m	0.00752	0.00928	0.01134	0.01434	0.0314	0.05139	0.06774
87m	0.00757	0.00933	0.0114	0.0144	0.03151	0.05159	0.06797
88m	0.00762	0.00938	0.01147	0.01447	0.03162	0.05178	0.06821
89m	0.00767	0.00943	0.01153	0.01453	0.03173	0.05198	0.06845
90m	0.00772	0.00948	0.01159	0.01459	0.03184	0.05217	0.06869
91m	0.00777	0.00953	0.01165	0.01466	0.03195	0.05237	0.06893
92m	0.00781	0.00958	0.01171	0.01472	0.03206	0.05258	0.06917
93m	0.00786	0.00963	0.01177	0.01478	0.03218	0.05278	0.06941
94m	0.00791	0.00968	0.01182	0.01484	0.03229	0.05299	0.06965
95m	0.00795	0.00973	0.01188	0.0149	0.03241	0.0532	0.06989
96m	0.008	0.00977	0.01193	0.01496	0.03253	0.05341	0.07013
97m	0.00805	0.00982	0.01199	0.01502	0.03264	0.05362	0.07037
98m	0.00809	0.00987	0.01204	0.01508	0.03276	0.05384	0.07061
99m	0.00814	0.00991	0.01209	0.01514	0.03289	0.05406	0.07085
100m	0.00819	0.00996	0.01214	0.0152	0.03301	0.05428	0.0711
101m	0.00823	0.01001	0.01219	0.01526	0.03313	0.05451	0.07134
102m	0.00827	0.01005	0.01224	0.01531	0.03326	0.05474	0.07158
103m	0.00832	0.0101	0.01229	0.01537	0.03339	0.05497	0.07183
104m	0.00836	0.01014	0.01234	0.01543	0.03352	0.05521	0.07208
105m	0.00841	0.01018	0.01238	0.01548	0.03365	0.05545	0.07232
106m	0.00845	0.01022	0.01242	0.01554	0.03378	0.0557	0.07257
107m	0.00849	0.01027	0.01247	0.01559	0.03392	0.05594	0.07282
108m	0.00854	0.01031	0.01251	0.01564	0.03406	0.0562	0.07307
109m	0.00858	0.01035	0.01255	0.0157	0.03419	0.05646	0.07332
110m	0.00862	0.01039	0.01258	0.01575	0.03434	0.05672	0.07357
111m	0.00866	0.01043	0.01262	0.0158	0.03448	0.05699	0.07383
112m	0.0087	0.01047	0.01266	0.01585	0.03463	0.05726	0.07408
113m	0.00874	0.01051	0.01269	0.0159	0.03477	0.05753	0.07434
114m	0.00878	0.01054	0.01272	0.01595	0.03492	0.05782	0.0746
115m	0.00882	0.01058	0.01275	0.016	0.03508	0.0581	0.07486
116m	0.00886	0.01062	0.01278	0.01605	0.03523	0.05839	0.07512
117m	0.0089	0.01065	0.01281	0.01609	0.03539	0.05869	0.07539
118m	0.00894	0.01069	0.01283	0.01614	0.03555	0.059	0.07565
119m	0.00897	0.01072	0.01286	0.01619	0.03571	0.05931	0.07592
120m	0.00901	0.01075	0.01288	0.01623	0.03588	0.05962	0.07619

Table 3: The credit spread data file

B References

YASHKIR CONSULTING. (<http://www.yashkir.com/page.php?11>).